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On resource depletion and productivity: The case of the Chilean copper industry

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ABSTRACT

How resource depletion affects productivity is a crucial question for several industries. In fact, several natural resource-exporting countries have seen their productivity levels affected by resource depletion. Nevertheless, usually, it is not clear what the real productivity growth is, without discarding the effects of resource depletion in the production structure. The main aim of the paper is to empirically answer a relevant issue regarding the Chilean copper mining industry, which is, the slowdown of its productivity in the last decade, considering in the analysis the role of resource depletion. In particular, we consider resource depletion to be an exogenous and unpaid force that opposes technological change and hence increases costs through time, capturing in this way some stylized facts of, for example, the mining and fishing industries. The decomposition framework was applied to the Chilean copper mining industry, one of the most important in the world, using data from the period of 1985–2015. The econometric results were robust and pointed to the fact that the productivity fell sharply during the period; however, it did not fall as much as the traditional estimation methods pointed out. Our model showed that as much as 15% of this decline was due to the increase of the resource depletion variable (copper ore grade).

1. Introduction

The growth of productivity is a crucial factor for economic sectors and countries to be sustainable. Total factor productivity (TFP) measures the share of economic growth that cannot be explained by increases in hours worked, increases in the amount of physical capital, or increases in the amount of human capital. In other words, TFP captures increases in efficiency and efforts to produce more with the same inputs. According to classical and neoclassical growth theories, it is not possible to sustain profits or Gross domestic product (GDP) growth rates without increases in productivity. In fact, in these models, the only source of per-capita growth in the long run is productivity, according to Solow (1956, 1957).

On the other hand, natural resource depletion is a reality for several renewable and non-renewable resource industries. Indeed, at the beginning of the past century, Hotelling (1931) already pointed out the inadequacy of static equilibrium in an industry in which the indefinite maintenance of a steady rate of production is a physical impossibility, and that is therefore bound to decline. As a result of this lower level of productivity, it would be expected that resource (commodity) prices will be permanently higher in the future.

In this context, how resource depletion affect productivity is a very important question for a variety of industries based on natural resources exploitation. In fact, lately, several natural resources-exporting countries have seen their national productivity levels affected due to resource depletion. For instance, over the past 20 years, production costs have increased in mining operations, causing productivity to fall, according to Tilton (2014). Thus, countries such as Australia, Canada and Chile have experienced low TFP growth rates (see Bradley and Sharpe, 2009; Capeluck, 2016; Syed et al., 2015; Topp et al., 2008; Topp and Kulys, 2014; Zheng and Bloch, 2014; Magendzo and Villena, 2012 and Bitran et al., 2014). One of the suspects in this situation is precisely the resource depletion that is measured in this industry as the ore grade of the mine.

This decline in mining productivity raises the question of whether technical change can offset the effects of resource depletion to have the chance of increasing growth rates over time. Nevertheless, usually, it is not clear what the real productivity growth is, without discarding the effects of resource depletion. The main aim of the paper is to empirically answer a relevant issue regarding the Chilean copper mining industry, which is, the slowdown of its productivity in the last decade, considering in the analysis the role of resource depletion. In particular, we consider resource depletion to be an exogenous and unpaid force that opposes technological change and hence increases costs through

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time, capturing in this way some stylized facts of, for example, the mining and fishing industries.

Our approach follows the existing literature, taking on the view that resource depletion in mining is not an input of production but rather an exogenous and unpaid variable, that allows us to connect directly resource depletion and productivity. Conceptually, an input of production usually has a price and quantity, and it is seen as part of the firm maximization's problem, where the manager identifies the optimal recipe for production, including the input amounts required. Clearly, this is not the case for resource depletion because a greater ore grade cannot be bought, and it cannot be modified in the short run by the managers; in fact, managers have to learn how to maximize profits considering the level of ore grade as a given exogenous and unpaid variable.[1]

In this paper, we present a model in line with those proposed by Rodríguez and Arias (2008) and Zheng and Bloch (2014). Nevertheless, there are some distinctive features in theoretical and empirical approaches. Firstly, we use a long-run cost function with capital, labour and energy as inputs, and technology and resources as exogenous unpaid variables. Certainly, our formulation is simpler and arguably more correct for the copper industry, since capital flows and technology are more flexible compared to other mineral resources. Secondly, our theoretical model considers the logarithm of the resource depletion index, which allows us to identify an explicit formulation of the elasticity of total costs and resource depletion, making easier the interpretation of the results. Thirdly, in our work, resource depletion is measured by copper ore grade, which is more suitable for the context of productivity analysis. Indeed, low levels of reserves do not necessarily are related with production costs and, therefore, do not always have an impact on the dynamics of productivity. Ore grade corresponds to the average percentage of pure mineral in the material moved. The main task of the operation is extracting the pure mineral, in this case copper, from that moving material. Thus, a low ore grade has a direct impact on production costs and, therefore, on productivity. Besides, the average of ore grade is usually used as the relevant resource index in the industry. Additionally, it is expected that, if the industry is competitive, as it is the Chilean case, the ore grade level will converge among firms, since firms with greater average cost usually goes out of business. Finally, an index of net productivity in mining in the context of resource depletion is introduced, which allow us to see the rate at which technical change must be increased so that the same level of costs can be maintained when the resource depletion factor is increased.

The papers is structured as follows. In the next section, the Chilean copper mining industry is briefly introduced. In Section 3 the related literature is discussed. In Sections 4 and 6, the theoretical and econometric models are introduced. Data and results are shown in Section 7. Finally, the conclusions and policy recommendation are presented in Section 8.

2. The Chilean copper mining industry

The decomposition framework developed in this research was applied to the Chilean copper mining industry. Chile is a mining country and always has been. Currently, Chile is an important producer of molybdenum, silver, gold and rhenium, and it is the largest producer of copper in the world. Additionally, Chile also holds the more important reserves of copper in the world; see Northey et al. (2014). In the 1960s and 1970s, Chile's copper exports accounted for more than 65% of Chilean exports but only about 8.5% of the GDP. In contrast, for the period of 2010–2014, copper exports reached 53% of the country's total exports and 16.4% of its GDP. Thus, even though Chilean exports have diversified, the relative importance of copper in the economy has increased. Chile's share of world copper reserves is around 28%; however, its share of world production has been falling, from 35% in 2000–30% in recent years. Undoubtedly, the decrease in the ore grade of the deposits partly responsible for this drop in production, as it costs more effort to extract the mineral.

On the other hand, copper tax revenues have shown significant variation in recent years. Current tax revenues are given by Codelco, a state-owned company, and private mining taxation. At the beginning of 2000, less than 10% of the central government's actual revenues were paid by mining taxes, reaching a peak of almost 35% in 2006, and now representing, again, around 10%, specifically 9.1% in 2014. In the same way, at the beginning of the 1990s, Codelco represented around 80% of national copper production. Today, however, the state's production reaches less than a third of national production. In terms of effective tax revenues, Codelco contributed US$ 3910 million in the period of 2001–2014, whereas revenues from private mining averaged US$ 2664 million in the same period. However, if we look at 2014, only 4.5% of tax revenue comes from Codelco, whereas 4.7% comes from private mining.

Based on Fig. 1, several stylized facts on the Chilean copper mining industry will be discussed. First, the upper-left figure shows us that increases in labour productivity took off when prices were low, and then, as prices went up, by the year 1990, labour productivity stabilized. More recently, labour productivity felt in line again with the felt in prices.

Garcia et al. (2001) and Garcia et al. (2000) focused on the causes of the jump in labour productivity in the Chilean copper industry during the 1990s. In particular, it separated the contribution of improvements at existing mines from the contribution of the other privately owned mines that came on stream during the 1990s. The conclusion of the work was that although the contribution of the new mines was somewhat greater than productivity improvements at existing mines, both were important. Regarding the same issue, Jara et al. (2010) concluded that innovation and new technology played an important role in increasing Chile's labour productivity. Without these improvements, the authors suggested that "*…many of Chile's older mines would no longer be in operation, Codelco would not be the world's largest copper producer, and copper exports from Chile would be about a third below their current level.*"

In the upper-right figure, we can clearly appreciate that investment increased sharply during the period; in general, it could be suggested that investments have followed the price evolution. In the lower-left graph, we see that production has tried to follow prices, but during the past years, it has been extremely consistent, specifically since 2003. Finally, in the lower-right figure, we can see that the relationship between production and the ore grade is inversely proportional. In other words, the more resources one gets, lower the amount of mineral that is obtained by unit of processed material, which clearly has a negative impact on the total costs of production.

From the above analysis, we can conclude that extremely important increases in prices during the period of analysis triggered important increases in the inputs of production. In this way, Chilean copper production has moved in the direction of mineral pricing, as expected. Nevertheless, there are two facts worthwhile to mention. First, production has started to slow down despite the increases in inputs, and second, the ore grade has descended greatly in the past years as production has increased.

Theses facts suggest that to obtain long-term growth rates, the productivity of the Chilean mining industry must enough to compensate for the higher costs, which implies resource depletion in total factor productivity. The main policy question in this scenario is whether technical change can offset the effects of resource depletion to have the chance of increasing growth rates over time. This issue will be addressed via a theoretical model and several econometric estimations in the next sections.

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[1] The ore grade of a mine can be changed, changing drastically the production technology, which requires important levels of investments, as is the case of the new large-scale underground mining projects.

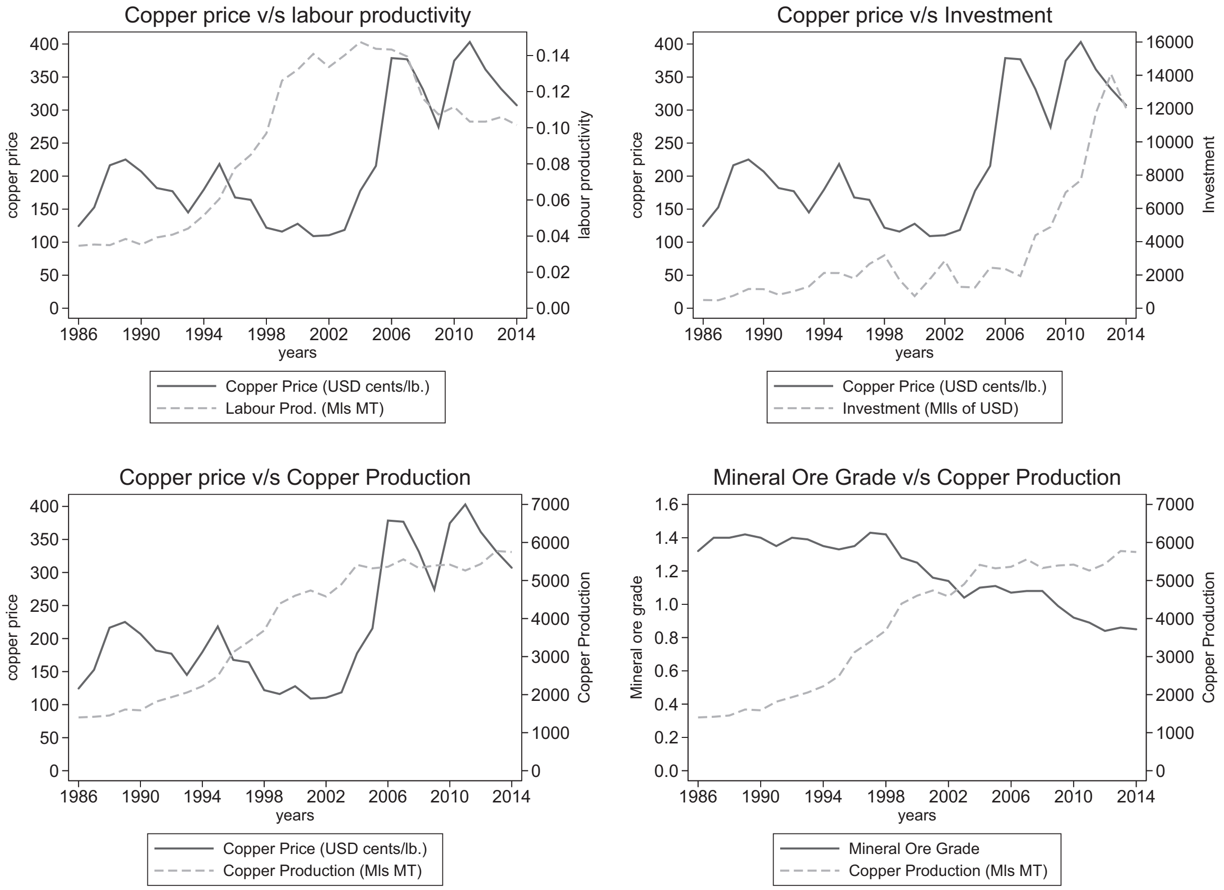

Fig. 1. Copper Price versus Labour Productivity, Investment, Production and the Ore Grade of Copper versus Production in Chile 1986–2014.

3. Related literature

There is not an extensive literature on the effects of natural resource depletion on productivity. Furthermore, there is not a clear consensus on how to deal with the effects of natural resource depletion on productivity. Basically, the main approach to adjusting the productivity measurement for natural resource depletion is considering resource depletion as another input in the production function. In addition, the analysis is divided into: i) growth accounting exercises and ii) econometric studies based on cost functions.

To our knowledge, the first academic attempt to measure the effects of natural resource depletion on productivity is Wedge (1973). It uses an index of the ore grade of a mineral in the context of a growth accounting framework, as a substitute for natural resource inputs in Canadian mining. The author found a measurable increase in the rate of productivity growth due to the correction for the natural resource factor.

Later on, Lasserre and Ouellette (1988, 1988) and Lasserre and Ouellette (1991), also using the growth accounting approach, suggested a method for computing simple Divisia indices of TFP for resource industries. In particular, the authors included the resource input as an explicit factor in the mining production function and also used the changes in ore grade to approximate the changes in the quality of the resource input.

Topp et al. (2008), using the growth accounting approach, estimated an index of profitability that adds grade changes; gas and mineral oil flows; and the ratio of salable to raw coal in the Australian mining industry. The estimated TFP growth in Australian mining was 2.5% per year on average over the growth period: 1975–2007. These results were very different from the conventional TFP measured growth of 0.01% in the same period. Using the growth accounting approach, Zheng and Bloch (2010) considered the period between the years of growth of 1975–2007 and found that the TFP changed from 0.01% to 1.15% when changes of natural resource inputs were added.

Caselli and Feyrer (2007) computed the output share of natural resources factor used in the analysis. This estimation was done using macroeconomic data for several countries. Even if their work did not consider a productivity analysis, their results remarked the importance of considering natural resources, differentiating them from the human-derived capital stock. Thereby, their results are relevant within the discussion of natural resources in the growth accounting computation.

More recently, Monge-Naranjo et al. (2015) constructed a new database to measure the income shares of natural resources for many countries and years. In their analysis, they explicitly considered natural resources to be inputs of production and measured their aggregate rents. The results are in line with Caselli and Feyrer (2007) with regards to the importance of natural resource differentiation. In addition, the authors argued that this is most severe for lower-income countries, which tend to have higher natural resource shares in aggregate incomes.

De La Huerta and Luttini (2017), following both former works, calibrated the contribution of exhaustible natural resources to the economy value added. Considering a analysis with natural resources, they studied the declining growth rate of the Chilean mining sector's added value since the year of 2000. To measure the effect on an aggregate country level, they used the framework proposed by Bernard and Jones (1996), where sector TFPs are compounded to achieve an aggregate TFP measure. Their results are in favor of those mentioned in Monge-Naranjo et al. (2015).

All of the studies above were conducted with the goal of adjusting the overlooked role of the exhaustible resources in the standard framework. Therefore, systematic weaknesses in the conventional growth accounting measure had to be considered in the analysis. Barro (1999) presented those issues affecting the interpretation of the Solow residual as a measure of technological change (Solow, 1957). In particular, its computation assumes that the social marginal products can be measured by observed factor prices. This assumption is fulfilled only under a particular scenario of competitive markets and long-run equilibrium. In reality, this is quite a difficult assumption to find in practice. Indeed, the assumptions of competitive markets and long-run equilibrium are not reasonable assumptions in a natural resource industry.

Another approach that allows for accounting for resource depletion, and allows to mitigate the former weaknesses, is the dual measure of TFP. The dual cost approach estimate of the TFP growth rate uses the same factor-income shares as the primal production approach (conventional calculation) estimate does but considers changes in factor prices, rather than quantities. The intuition for the dual estimate on the right-hand side of the equation is that rising factor prices (for factors of given quality) can be sustained only if the output is increasing for given inputs. Therefore, the appropriately weighted average of the growth of the factor prices measures the extent of TFP growth.[2]

In this line of research, Stollery (1985) calculated TFP trends in seven Canadian mining industries and related changes in productivity to various factors, including the quality of the ore being mined. He found that the productivity decline in mining was mainly due to the fall in the mineral grade, the contraction of the output, increases in interest rates and an apparent decline in the rate of technical innovation.

Rodríguez and Arias (2008) proposed a dual estimation of the Solow Residual. In particular, the authors provided a new decomposition of the Solow Residual for extractive industries in which the level of reserves is likely to affect extraction costs. Using data from the coal mining industry in Spain from the period of 1975–2001, they provided a dual estimation of the Solow Residual, which considers the following inputs: employment, energy, materials, capital and natural resources. To measure the resource depletion, they use the mineral reserves in Tons. They used a variable cost function and capital as a quasi-fixed input. The main result was that the depletion of natural resources requires an annual increase of input use of 1.29%, which is the measured effect of the depletion of natural resources over the Solow Residual. Rodríguez et al. (2015), also using data on slate mining in Galicia (Northern Spain), and with the same economic model, found that non-renewable natural resources account for the increase of the total cost growth in less than 1.5% per year on average.

The same approach of Rodríguez and Arias (2008) was used by Zheng and Bloch (2014) for the Australian mining industry, considering resource depletion to be an input of production in the context of a variable cost function and capital as a quasi-fixed input. To measure the resource depletion, they use the mineral reserves in Tons. The results showed that the average TFP growth in Australian mining based on the dual cost-function measure of technical change is 2% over the sample period 1974–1975 to 2007–2008, rather than −0.2% from the published index. The difference arises because declining natural resource inputs, the effects of capacity utilization and returns to scale have all reduced the true TFP growth.

In our work, we present a model in line with those proposed by Rodríguez and Arias (2008) and Zheng and Bloch (2014). Nevertheless, there are some distinctive features in our theoretical and empirical model that we discuss below.

Firstly, the studies mentioned above use a variable cost function. In the case of Zheng and Bloch (2014), their study only contains one variable input, namely labour, while both capital and resource inputs are treated as fixed inputs in the short run. Rodríguez and Arias (2008) use labour, energy, materials, and a quasi-fixed input, capital, the technology, and a level of reserves of the natural resource. In contrast our paper, we use a long-run cost function with capital, labour and energy as inputs, and technology and resources as exogenous unpaid variables. Certainly, our formulation is simpler and arguably more correct for the copper industry, since capital flows and technology are more flexible compared to other mineral resources.

Secondly, our theoretical model considers the logarithm of the resource depletion index, which allows us to identify an explicit formulation of the elasticity of total costs and resource depletion. Certainly the calculation of the elasticity facilitates the interpretation of the problem, shedding light to the issue at hand, namely the importance of resource depletion in the evolution of productivity in the Chilean mining industry.

Thirdly, resource depletion is also considered differently in each paper. Rodríguez and Arias (2008) use an annual time series of reserves to control for the effect of resource depletion for the Spanish coal industry. While (Zheng and Bloch, 2014) consider the part of productive capital stock in mining that is attributed to mineral and petroleum exploration as a proxy for the services derived from the stock of resource inputs. Thus (Rodríguez and Arias, 2008) basic assumption is that coal extraction is the main force in the evolution of reserves while new discoveries are not very relevant. They claim that this assumption is not unrealistic in the case of coal mining in Spain. Zheng and Bloch (2014) estimation implies that the resource inputs used are subject to declining productivity over time, similar to depreciation on the conventional physical capital stock. In our work, resource depletion is measured by copper ore grade, which is more suitable for the context of productivity analysis. Indeed, low levels of reserves not necessarily are related with production costs and, therefore, do not always have an impact on the dynamics of productivity. Ore grade corresponds to the average percentage of pure mineral in the material moved. The main task of the operation is extracting the pure mineral, in this case copper, from that moving material. Thus, a low ore grade has a direct impact on production costs and, therefore, on productivity. Besides, the average of ore grade is usually used as the relevant resource index in the industry. Additionally, it is expected that, if the industry is competitive, as it is the Chilean case, the ore grade level will converge among firms, since firms with greater average cost usually goes out of business. Finally, an index of net productivity in mining in the context of resource depletion is introduced, which allow us to see the rate at which technical change must be increased so that the same level of costs can be maintained when the resource depletion factor is increased.

4. Theoretical model

Here, we will follow the standard decomposition of the TFP growth procedure, see (Denny et al., 1981). In the following table the notation of the model is defined.

The growth rate of TFP is defined as:

$$T\dot{F}P = \dot{Q} - \dot{F} \tag{1}$$

where Q is the output and F represents the total factor input, and a dot denotes a rate of growth (as variables are in a logarithm). For F, the conventional Divisa index is used:

$$\dot{F} = \sum_i S_i \dot{X}_i \tag{2}$$

where S_i is the cost share of variable input i, X_i is the quantity of input i and $\dot{X}_i$ represents the growth rate of X_i. $S_i \equiv \frac{P_i X_i}{C}$, where P_i is the price of input i, and C denotes total cost.

It is important to keep in mind that when cost-minimizing behaviour is assumed, it is well known that the dual cost function provides an equivalent description of the technology of any production function considered. Let us assume now a cost function that considers natural resource depletion:

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[2] As Barro (1999) argues, the dual estimation only uses the condition $Y = RK + wL$. No assumptions are made about the relations of factor prices to social marginal products or about the form of the production function.

$$C = f(P_i, Q, R, t) \tag{3}$$

where P is the price, Q is the output, R is an index of resource depletion, and t is an index of "*technology*", which is a time function. Taking the logarithmic time derivative of Eq. (3):

$$\frac{dC}{dt} = \sum_i \frac{\partial C}{\partial P_i}\frac{dP_i}{dt} + \frac{\partial C}{\partial Q}\frac{dQ}{dt} + \frac{\partial C}{\partial R}\frac{dR}{dt} + \frac{\partial C}{\partial t} \tag{4}$$

We can rewrite Eq. (4) as:

$$\dot{C} = \sum_i S_i\dot{P}_i + \varepsilon_{cq}\dot{Q} + \varepsilon_{cr}\dot{R} + \varepsilon_{ct} \tag{5}$$

where $S_i = \frac{\partial lnC}{\partial lnP_i}$ is the cost share of variable input i, $\varepsilon_{cq} = \frac{\partial lnC}{\partial lnQ}$ is the total cost with respect to output elasticity, $\varepsilon_{cr} = \frac{\partial lnC}{\partial lnR}$ is the total cost with respect to natural resource depletion elasticity and $\varepsilon_{ct} = \frac{\partial lnC}{\partial t}$ is the rate of technical change.

Here, technical change is defined as any shift in the production frontier, thus measuring the relative change in output Q due to the partial effect of technology index t. A Hicks-neutral technical change occurs when the marginal rate of substitution (MRS) between any two inputs is unaffected by technical change. A Hicks-neutral technical change is equivalent to a homothetic in the isoquants.

On the other hand, as noted by Ohta (1974), the total cost and its time derivative are defined as:

$$C = \sum_i P_i X_i \tag{6}$$

$$\dot{C} = \sum_i S_i\dot{P}_i + \sum_i S_i\dot{X}_i \tag{7}$$

Equating Eqs. (1) and (2) yields:

$$\sum_i S_i\dot{X}_i = \varepsilon_{cq}\dot{Q} + \varepsilon_{cr}\dot{R} + \varepsilon_{ct}$$

Replacing Eqs. (1) and (2), we get:

$$T\dot{F}P = \dot{Q}(1 - \varepsilon_{cq}) - \varepsilon_{cr}\dot{R} - \varepsilon_{ct} \tag{8}$$

In the next section, Eq. (8) is estimated econometrically for the Chilean copper industry.

5. Technical change and resource depletion

In this setting, the ore grade of the mine is defined as the proportion of the rock that actually contains valuable metal that can be extracted from it. It is usually expressed as a percentage, and it constitutes an exogenous variable, since it cannot be easily changed by the mineral producer, and to some extent it is a stochastic variable in nature. Obviously, the lower the ore grade the greater the extraction costs, because firms needs to extract more rock and materials for smaller quantities of valuable mineral for unit of volume, in other words $\frac{\partial C}{\partial r} < 0$. On the other hand, since firms minimize cost, they plan their operations trying to exploit first the part of the mine with the greater ore grade. In fact, this task in mining engineering is called mine planning and usually considers: i) strategic and tactic components (short, medium and long-term), and ii) the following variables into the analysis (besides economic factors): geological and geotechnical models, extraction strategies, metallurgical variables, and the different loss factors. In this context, we can safely assume that the ore grade is always a decreasing variable (at least for the same operation and without considering large investments). Thus, the resource depletion is modeled as an exogenous and unpaid force that effectively offsets gains from technological change and hence increase costs through time.

In order to have a quantitative idea of this problem, let's take a derivative of Eq. (3) with respect to the resource depletion index:

$$\frac{dC}{dR} = \sum_i \frac{\partial C}{\partial P_i}\frac{dP_i}{dR} + \frac{\partial C}{\partial Q}\frac{dQ}{dr} + \frac{\partial C}{\partial t}\frac{dt}{dR} + \frac{\partial C}{\partial R} \tag{9}$$

Assuming that resource depletion in this particular mining production will not affect the market price so that $\frac{dP_i}{dR} = 0$, and assuming as well that the resource depletion is not yet affecting the capacity utilization of the plant so $\frac{dQ}{dR} = 0$, we can rewrite Eq. (9) as:

$$\frac{dC}{dR} = \frac{\partial C}{\partial t}\frac{dt}{dR} + \frac{\partial C}{\partial R} \tag{10}$$

The most interesting practical question in terms of mining economics is whether the technological change will off-set the resource depletion effect. The answer to this question allows, for example, for the manager to infer the useful life of the business, or in other words, the sustainability of the mining operation. We can model that scenario, asking, what should be the technological change that allows us not to increase costs due to resource depletion?, so that $\frac{dC}{dR} = 0$. From Eq. (10) we get:

$$\rho = \frac{dt}{dR} = -\frac{\partial C}{\partial R}\Big/\frac{\partial C}{\partial t} \tag{11}$$

Eq. (11) shows us the rate at which technical change must be increased so that the same level of cost can be maintained when the resource depletion factor is increased. We call it, ρ, the *net productivity index*. In technical terms, this is known as the marginal rate of technical substitution (MRTS), in this case, between resource depletion and technical change considering cost instead of production. A textbook recipe will tell us that when relative input usages are optimal, the slope of the isoquant at the chosen point equals the slope of the isocost curve. In general, if $\rho > 1$ then the net mining technical change index would be with negative impact since $\frac{\partial C}{\partial R} > \frac{\partial C}{\partial t}$.

From the mineral producer point of view, the question is how much I must invest in new technology in order to get the same level of costs that when I face increasing levels of resource depletion. It is important to point out that this is exactly the point of discussion for example in the Chilean mining industry today. Whether to invest billions of dollars and go deeper underground to get better ore grades or to close operations or open new ones with higher levels of ore grade, which are much more smaller.

From a more aggregated perspective, for example when you measure technical change and productivity for the mining sector of a whole country, we should have to consider this measure of net productivity, in order to not confuse the resource depletion effect with a decrease in productivity, see (Tilton, 2014).

From a empirical perspective, the net productivity index can be calculated as:

$$\rho = -\frac{\partial lnC}{\partial lnR}\Big/\frac{\partial lnC}{\partial t} = \frac{\varepsilon_{cr}}{\varepsilon_{ct}} \tag{12}$$

6. Econometric strategy

In our model, the cost function has as arguments the level of output, input prices, and an index variable of natural resource depletion. To develop an econometric model, we will use a translog function. In particular, the translog cost function is a second-order Taylor's series approximation in logarithms to an arbitrary cost function (see Christensen et al., 1973). The main attribute of translog cost functions is that they impose no prior restriction on the production structure, namely: homotheticity, homogeneity, constant returns to scale, or unitary elasticities of substitution. In fact, they allow for testing all of these alternative production configurations. The translog cost function that we will use is:

$$\begin{aligned} ln\left(\frac{C}{P_E}\right) &= \theta_0 + \theta_Q lnQ + \frac{1}{2}\theta_{QQ}(lnQ)^2 + \theta_r lnr + \frac{1}{2}\theta_{rr}(lnr)^2 + \theta_{Qr} lnQ\cdot lnr + \\ &+ \theta_t\cdot t + \frac{1}{2}\theta_{tt}\cdot t^2 + \theta_{tQ} t\cdot lnQ + \delta_K ln\left(\frac{P_K}{P_E}\right) + \delta_L ln\left(\frac{P_L}{P_E}\right) \\ &+ \frac{1}{2}\gamma_{KK}\ln\left(\frac{P_K}{P_E}\right)\ln\left(\frac{P_K}{P_E}\right) + \gamma_{KL}\ln\left(\frac{P_K}{P_E}\right)\ln\left(\frac{P_L}{P_E}\right) + \frac{1}{2}\gamma_{LL} ln\left(\frac{P_L}{P_E}\right)\cdot ln\left(\frac{P_L}{P_E}\right) \\ &+ \gamma_{tK} t\cdot ln\left(\frac{P_K}{P_E}\right) + \gamma_{tL} t\cdot ln\left(\frac{P_L}{P_E}\right) \\ &+ \gamma_{QK} lnQ\cdot ln\left(\frac{P_K}{P_E}\right) + \gamma_{QL} lnQ\cdot ln\left(\frac{P_L}{P_E}\right) \\ &+ \gamma_{rK} lnr\cdot ln\left(\frac{P_K}{P_E}\right) + \gamma_{rL} lnr\cdot ln\left(\frac{P_L}{P_E}\right) + \mu_t \end{aligned} \tag{13}$$

The estimation of Eq. (13) can be carried out directly; however, as the capital, labour and energy shares sum to unity ($\sum_{i=K,L,E} S_i = 1$), only the two cost-share equations are needed to estimate some of the required parameters.[3] In this case, TFP estimations requires all three Eqs. (13), (14), and (15) to estimate all the coefficients.

$$S_K = \frac{\partial lnC}{\partial lnP_K} = \delta_K + \gamma_{KK} ln\left(\frac{P_K}{P_E}\right) + \gamma_{KL}\ln\left(\frac{P_L}{P_E}\right) + \gamma_{tK} t + \gamma_{QK} lnQ + \gamma_{rK} lnr \tag{14}$$

$$S_L = \frac{\partial lnC}{\partial lnP_L} = \delta_L + \gamma_{KL} ln\left(\frac{P_K}{P_E}\right) + \gamma_{LL}\ln\left(\frac{P_L}{P_E}\right) + \gamma_{tL} t + \gamma_{QL} lnQ + \gamma_{rL} lnr \tag{15}$$

One of the most common techniques for estimating a system of simultaneous equations in econometrics is the seemingly unrelated regression (SUR). It is well known that this estimation procedure is more efficient OLS when the errors across the equations in the system are contemporaneously correlated, see Berndt (1990). Here, SUR is used to estimate jointly the coefficients in Eqs. (13), (14), and (15). As usual, the errors associated with the two share equations are assumed to be normally distributed. The parameter estimates using the iterative SUR are numerically equivalent to those of the maximum likelihood estimation procedure.

Once all parameters are estimated, a series of elasticities and the decomposition of TFP growth can be calculated. Thus, according to Eq. (8), in order to estimate the growth rate of TFP, we have to calculate $\dot{Q}$, $\dot{R}$, ε_{ct}, ε_{cr}, ε_{cq}. $\dot{Q}$ and $\dot{R}$ are easier, since they are just the rate of growth of Q and R respectively. On the other hand, $\varepsilon_{ct} = \frac{\partial lnC}{\partial t}$, $\varepsilon_{cr} = \frac{\partial lnC}{\partial lnR}$, and $\varepsilon_{cq} = \frac{\partial lnC}{\partial lnQ}$. Here, in order to estimate every elasticity, we have to logarithmically differentiates Eq. (13) with respect to t, lnR, lnQ, respectively, see for instance Berndt (1991).

Firstly, the rate of technical change is estimated as:

$$\varepsilon_{ct} = -\left(\theta_t + \theta_{tt}\cdot t + \theta_{tQ} lnQ + \gamma_{tK} ln\left(\frac{P_K}{P_E}\right) + \gamma_{tL} ln\left(\frac{P_L}{P_E}\right)\right) \tag{16}$$

$\varepsilon_{ct} = -(\partial lnC/\partial t)$ does not measure the net technological effect. The effect of scale economies is included in this variable. It must thus be divided by ε_{cr} to calculate the net effect of technical change. A particular case is given when the underlying production technology is characterised by a constant return to scale.

Second, the total cost with respect to natural resource depletion elasticity is computed as:

$$\varepsilon_{cr} = \theta_r + \theta_{rr} lnr + \theta_{Qr} lnQ + \gamma_{rK} ln\left(\frac{P_K}{P_E}\right) + \gamma_{rL} ln\left(\frac{P_L}{P_E}\right) \tag{17}$$

Finally, the total cost with respect to output elasticity ε_{cq} is defined as:

$$\varepsilon_{cq} = \theta_Q + \theta_{QQ} lnQ + \theta_{Qr} lnr + \theta_{tQ} t + \gamma_{QK} ln\left(\frac{P_K}{P_E}\right) + \gamma_{QL} ln\left(\frac{P_L}{P_E}\right) \tag{18}$$

Based on Hanoch (1975), economies of scale are defined as one minus the elasticity of total cost with respect to output:

$$\Psi = 1 - \varepsilon_{cq} \tag{19}$$

Ψ is positive for scale economies and negative for diseconomies of scale. On the other hand, returns to scale (RTS) can be computed as the inverse of the elasticity of total cost with respect to output:

$$\mu = 1/\varepsilon_{cq} \tag{20}$$

Both returns to scale and economies of scale are the same when the production function is homothetic.

Other very useful elasticities are the cross-price elasticities of the factor demand, which can be calculated as:

$$\eta_{ii} = \frac{\gamma_{ii}}{S_i} + S_i - 1 \tag{21}$$

$$\eta_{ij} = \frac{\gamma_{ij}}{S_i} + S_i, \text{ for } i \neq j \tag{22}$$

In addition, the Allen partial elasticities of substitution between two factors i and j, Uzawa (1962), are:

$$\sigma_{ij} = \frac{\gamma_{ij}}{S_i S_j} + 1, \text{ for } i \neq j \tag{23}$$

These elasticities measure the percentage of change in factor proportions due to a one-percent change in their relative prices. In general, $\eta_{ij} = S_i\sigma_{ij}$.

7. Results and discussion

Our econometric analysis is based on the Chilean mining industry during the period of 1986–2015. The variables used in the analysis and their sources are detailed in Table A.1 in the Appendix. The descriptive statistics of the variables are presented in Table A.2, also in the Appendix.

First, the cost share equations are modeled using the non-linear SUR method and the maximum likelihood estimation Table 1. The results for Eqs. (14) and (15) are presented in Table 2. In particular, we analyze the econometric results for the share equations with and without considering resource depletion. As we already show in Table A.1, the proxy for resource depletion is the ore grade of the mineral in the Chilean copper industry. We can clearly see that both models are quite robust with all variables significant at 1%. Resource depletion variables are also significant, meaning that the resource depletion elasticity increase when the ore grade of the mineral decreases see Eq. (17).

These results indicate that the effects of resource depletion are much greater in the capital input that in the labour input. Thus, the resource depletion variable is included in the overall fit of the model improves, specially for the capital share equation. This analysis confirms the fact that resource depletion is a necessary variable for describing production and cost structures in mining industries Table 3.

Because we need to consider cost function 13 to fully estimate the TFP growth and their determinants, in Table 5, the joint results of the system of equations formed by Eqs. (13), (14) and (15) are presented. Again, results are statistically robust, and all variables are significant at 1% with the exception of θ_{rt}.

With the results shown in Table 5, we can calculate the own- and cross-price elasticities; see Table 4. The own-price elasticities for capital and energy are negatives, indicating that increases in prices will

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[3] Let us keep in mind that is a standard procedure, in the context of statistical inference in equation systems, to drop an arbitrary equation in order to avoid the disturbance and residual cross-products matrices to be singular. Then the n-1 equation system could be estimated, see Chapter 9 in Berndt (1991), page 472. That is why Eqs. (13), (14), and (15) are divided by P_E and equation S_E is dropped.

Table 1
Nomenclature.

Variable	Definition	Variable	Definition
q	Product	τ_i	Technical change bias of factor i
q_k	Capital stock	ψ_i	Resource depletion bias on factor i
q_l	Employment	ε_{ct}	Rate of technical change
q_e	Energy	ε_{cr}	Natural resource depletion elasticity
p_l	Wages	ρ	Net mining technical change index
p_e	Energy price	ρ_i	Net effect of technical change for factor i
r	Ore grade	η_{ij}	Cross-price elasticity between i and j
		σ_{ij}	Partial elasticity of substitution between i and j

Table 2
Cost share equation regressions.

a) Without considering the resource depletion				
	Observations	Parameters	RMSE	R-square
sk	29	5	0.00894	0.70329
sl	29	5	0.00000	0.92504
	Coefficient	Err.	Z-value	P-value
ak	−0.43279	0.05823	−7.43297	0.00000
gkk	0.00938	0.00265	3.53721	0.00040
gkl	−0.00001	0.00000	−7.22426	0.00000
bkq	0.02406	0.00279	8.62965	0.00000
dkt	−0.00294	0.00045	−6.48024	0.00000
al	−0.00027	0.00005	−5.46411	0.00000
gll	0.00002	0.00000	10.81386	0.00000
blq	0.00001	0.00000	5.48319	0.00000
dlt	−0.00000	0.00000	−8.70612	0.00000
b) Considering the resource depletion				
	Observations	Parameters	RMSE	R-square
sk	29	6	0.00747	0.79254
sl	29	6	0.00000	0.93496
	Coefficient	Err.	Z-value	P-value
ak	−0.27614	0.05684	−4.85818	0.00000
gkk	0.01349	0.00258	5.23091	0.00000
gkl	−0.00001	0.00000	−6.85093	0.00000
bkq	0.01934	0.00261	7.41066	0.00000
bkr	−0.08386	0.02405	−3.48695	0.00049
dkt	−0.00407	0.00061	−6.63495	0.00000
al	−0.00025	0.00005	−4.88712	0.00000
gll	0.00003	0.00000	10.57513	0.00000
blq	0.00001	0.00000	5.00922	0.00000
blr	−0.00001	0.00000	−2.40619	0.01612
dlt	−0.00000	0.00000	−8.70507	0.00000

Table 3
Cost share and cost equation regressions.

	Observations	Parameters	RMSE	R-square
lnc	29	15	0.03515	0.99673
sk	29	5	0.00897	0.70122
sl	29	5	0.00000	0.92434
	Coefficient	Err.	Z-value	P-value
a0	−4.73480	0.00000	.	.
ak	−0.41699	0.05677	−7.34466	0.00000
al	−0.00032	0.00005	−6.81477	0.00000
gkk	0.01055	0.00260	4.05735	0.00005
gll	0.00003	0.00000	12.76951	0.00000
gkl	−0.00001	0.00000	−8.69262	0.00000
by	2.55909	0.05749	44.51425	0.00000
byy	−0.12223	0.00535	−22.85607	0.00000
bky	0.02359	0.00268	8.81476	0.00000
bly	0.00001	0.00000	6.78446	0.00000
dt	−0.20322	0.06795	−2.99071	0.00278
dtt	−0.00548	0.00056	−9.75034	0.00000
dkt	−0.00290	0.00044	−6.56417	0.00000
dlt	−0.00000	0.00000	−10.36889	0.00000
dyt	0.01551	0.00337	4.60983	0.00000

Table 4
Own- and cross-price elasticities.

	k	l	e
k	−0.937557	−0.000668	0.0944237
l		0.468236	0.0818599
e			−0.093397

Table 5
Decomposition of the TFP growth rate.

Period	Rate of technical change	Scale effect	Resource depletion effect	TFP growth rate
1986–1994	−0.0801	0.1072	0.0004	0.0276
1994–2003	−0.0549	0.2149	−0.0023	0.1577
2004–2015	−0.0413	0.0938	−0.0242	0.0284
1986–1989	−0.0881	0.3032	0.0100	0.2250
1990–1994	−0.0736	−0.0104	−0.0053	−0.0892
1995–1999	−0.0615	−0.0135	−0.0006	−0.0755
2000–2004	−0.0484	0.2558	−0.0040	0.2033
2005–2009	−0.0506	0.1371	−0.0198	0.0667
2010–2015	−0.0319	0.0504	−0.0285	−0.0100

decrease the demand in each input. The energy price elasticity is very inelastic, reaching −0.0093, whereas the capital price elasticity is also inelastic but not at the same level of −0.093, in fact, it is 10 times more elastic than the energy elasticity is.

Not surprisingly, the own-price elasticities for labour are positive, indicating that real wages have increased and the amount of labour as well. One of the three reasons for this is the copper price supercycle that generated lower levels of austerity in the management, and on the other hand, the opportunistic behaviour of unions that pushed for higher wages during the end of the period. Finally, in 2010, the labour legislation in Chile imposed firms to treat contractors with the same rights and wages as their counterparts in the firms, which increased labour costs heavily.[4]

In Table 5, the decomposition of the growth rate of the TFP is presented with two different temporal disaggregations. Dividing the period into three parts, we can see that the productivity growth rate started with an average of 2.7% in the period of 1986–1994. The TFP growth rate was much higher for the period of 1994–2003, reaching 15.7%. This period took place after the investments occurred during the 1990s and before the price boom (2004–2015). Finally, during and after the price boom, the TFP growth rate decreased importantly, reaching to 2.8%, similar to its levels of the first period considered in the analysis.

The main reason for the fall in the last period considered in the analysis is the rate of technical change with −4.1%, followed by the resource depletion effect with −2.4%.[5] In this period, productivity was supported by the scale effect that sum almost 1%. At the bottom of

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[4] This is not surprising in commodity based economies where the labour supply elasticity is usually very inelastic, especially after boom prices as was the case of copper. If the supply is inelastic, as was the case of Chilean mining during the last copper price boom, the quantity of labour supplied will not change much as wages change. Indeed, in simple terms, when the prices of the commodity are extremely high, there is a lot of pressure to produce more and more, but in order to produce more you need more labour (miners, drivers, cooks, etc). The problem is that the economy runs out of workers, and then wages must go up, in line with the demand for workers. In fact, it could be perfectly possible that both wages and quantity move in same direction and hence price elasticity comes out to be positive, which is consistent with our estimations.

[5] It is important to mention that the ore grade was multiplied by minus one to facilitate the interpretation of resource depletion. Thus, when the ore grade decreased, the resource depletion increased.

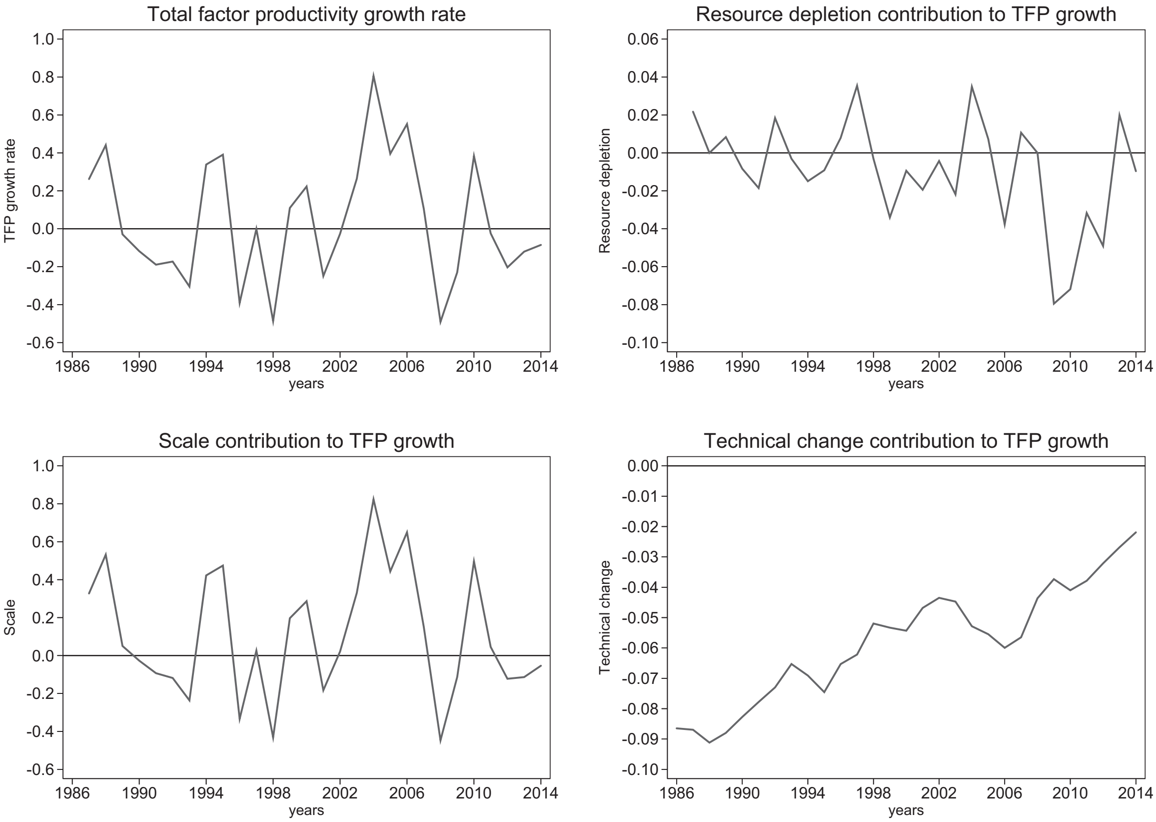

Fig. 2. Evolution of TFP growth rate and their determinants.

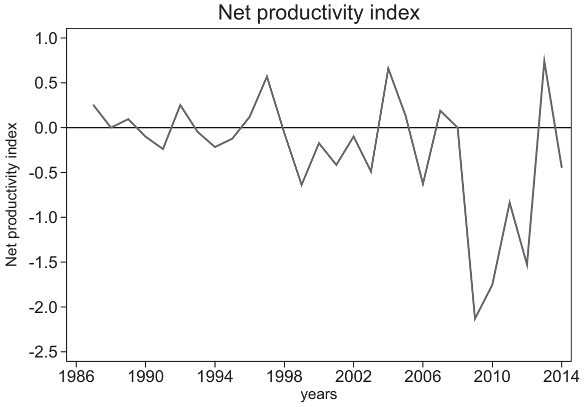

Fig. 3. Net productivity index in the copper industry in Chile.

Table 5, the decomposition of the TFP growth rate is shown in a more disaggregated manner.

Finally, in Fig. 2 the evolution of the TFP growth rate and its determinants is presented graphically for all of the period under analysis.

Three conclusions can be drawn from this analysis. First, the TFP in the Chilean copper industry has been mainly driven by the scale effect. The great variability of the scale effects, following to some extent the economic cycles, indicates that capacity utilization varied a great deal during the period. Indeed, production levels have an important effects on cost and productivity, suggesting that all installations have not been used fully. Important diseconomies of scale are shown.

Second, the resource depletion in the last part of the period had a significant impact on the TFP growth rate decay. In fact, our estimations showed that as much as 15% of the decline in the growth rate of TFP in the period of analysis was due to the increase of the resource depletion variable, represented in this case by the copper ore grade, which declined sharply in that period.

Third, contrary to what the labour productivity trend suggested previously, TFP mostly increased in the period of analysis; nevertheless, the rate of growth decreased markedly. In the same way, the technical change increased during the whole period too, even though its magnitude has always been negative. This point is quite relevant for the Chilean copper industry because it is frequently argued that the low productivity is given by the low level of technological change, which is only partially true. The main policy issue is that technical change, in fact, has not achieved to offset the effects of resource depletion, thus jeopardizing sustained long-term growth rates.

Finally, in this case, the evolution of the net productivity index for the period under investigation is presented Fig. 3. We can see that during the period 2009–2012 the net productivity index is greater than minus one, indicating that the technical change did not compensate the resource depletion. This unsustainable path was broke after 2013.

8. Conclusions

In this paper, we have analyzed the slowdown of the productivity of the Chilean copper mining industry in the last decade, considering in the analysis the role of resource depletion. In our model, TFP growth was linked to an index of resource depletion, economies of scale and technological change in the context of a translog cost function. In particular, we consider resource depletion to be an exogenous and unpaid force that opposes technological change and hence increases costs through time, capturing in this way some stylized facts of, for example, the mining and fishing industries.

The decomposition framework was applied to the Chilean copper industry, using data from the period of 1985–2015. The econometric results were robust and pointed to the fact that the TFP growth rate fell sharply in the period; however, as much as 15% of this fall in the period of analysis was due to the increase of resource depletion variable, represented in this case by the copper ore grade, which declined sharply in this period.

From a methodological point of view, our approach followed the existing literature, taking on the view that resource depletion in mining is not an input of production but rather an exogenous and unpaid variable, that allowed us to connect directly resource depletion and productivity. This formulation not only proved to be econometrically robust, improving the statistical significance of the cost function approach, but also improved our understanding of a very relevant issue regarding the Chilean copper mining industry, which is, the slowdown of its productivity in the last decade. Finally, an index of net productivity in mining in the context of resource depletion is introduced, which allow us to see the rate at which technical change must be increased so that the same level of costs can be maintained when the resource depletion factor is increased.

For future research in this particular case study, it is highly recommended to generate a variable of capacity utilization, which is quite easy to incorporate into the econometric estimation using the same theoretical framework.

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Appendix A. Appendices

Table A.1
Variables and data sources.

Var. Name	Variable	Definition/Construction	Source
q	Product	Gross Domestic Product in current US$. This series is published by the Central Bank.	Central Bank of Chile
q_k	Capital Stock	Net Capital Stock in current US$. The data used since 1996 is from the Central Bank. The backwards data were constructed investment data from the Chilean Copper Commission using perpetual inventory. We supposed a year depreciation of 7.0%.	Central Bank of Chile and and the Chilean Copper Commission
q_l	Employment	Employment (Yearly Average). This series is published by the Chilean Copper Commission	the Chilean Copper Commission
q_e	Energy	Energy consumption (units: kWh). Since 2005, we use data from the Chilean Copper Commission. Since the year 1991 until 2004 we use data published by the National Commission of Energy in its National Energy Balance. The backwards data are from Central Bank of Chile (2001) (units: kWh).	National Commission of Energy, Central Bank and the Chilean Copper Commission.
p_l	Wages	Nominal wages in current US$. It was constructed using the nominal wage index published by the the Chilean Copper Commission. We considered a base year in current US$ for every year.	the Chilean Copper Commission
p_e	Energy Price	Nominal energy price in current US$. This series correspond to the price of Nominal energy published by El Sistema Interconectado del Norte Grande (SING). Backwards from year 1991, the series is adjusted using data from Sistema Interconectado Central (SIC) (units: US$/kWh).	SING and SIC
r	Ore grade	Average ore grade	the Chilean Copper Commission

Table A.2
Summary statistics.

Variable	Mean	Std. Dev.	Min.	Max.	N
q	12.13	11.85	1.78	34.7	29
qk	31.99	28.15	11.89	108	29
ql	42.57	6.4	33.64	56.63	29
qe	12.49	6.33	3.1	23.13	29
pk	0.29	0.22	0.07	0.92	29
pl	188.61	118.99	25.21	425.03	29
pe	26.93	15.99	9.77	66.58	29
r	1.19	0.2	0.84	1.43	29

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